1 Mechanical vibrations of a material point

Problem 1: Mechanical vibrations of a material point are described by equation

$$y'' + 8y' + 17y = e^{-4t} \tag{1}$$

where the position of the point at the initial moment $t = 0$ and the moment $t = 1$ are given by

$$y(0) = 0, \quad y(1) = 0, \tag{2}$$

respectively. Determine the deviation of a point $y(t)$ from the equilibrium position at any moment in time t.

Solution: Let's find a general solution to a linear non-uniform equation. The characteristic equation for the corresponding homogeneous equation has roots $\lambda_{1,2} = -4 \pm i$, therefore we obtain the general solution of the homogeneous equation in the form:

$$y_0 = e^{-4t}\left(C_1 \cos t + C_2 \sin t\right) \tag{3}$$

Since the number -4 does not coincide with any of the roots of the characteristic equation, the particular solution will have the form

$$y_p = Be^{-4t}. \tag{4}$$

Finding

$$\begin{aligned} y_p' &= -4Be^{-4t} \\ y_p'' &= 16Be^{-4t} \end{aligned} \tag{5}$$

and substituting the results into the original equation, we get

$$16Be^{-4t} - 32Be^{-4t} + 17Be^{-4t} = e^{-4t}, \tag{6}$$

from which it follows $B = 1$ so

$$y_p = e^{-4t} \tag{7}$$

Therefore, the general solution to the original equation

$$y(t) = y_0 + y_t = e^{-4t}\left(1 + C_1 \cos t + C_2 \sin t\right) \tag{8}$$

Let us substitute the boundary conditions $t = 0$, $y = 0$ and $t = 1$, $y = 0$

$$\begin{cases} 1 + C_1 + 0 = 0 \\ e^{-4}(1 + C_1 \cos 1 + C_2 \sin 1) = 0 \end{cases} \tag{9}$$

Solving the system, we obtain

$$\begin{cases} C_1 = -1 \\ C_2 = \frac{\cos 1 - 1}{\sin 1}. \end{cases} \tag{10}$$

Finally,

$$y(t) = e^{-4t}\left(1 - \cos t + \frac{\cos 1 - 1}{\sin 1} \sin t\right). \tag{11}$$

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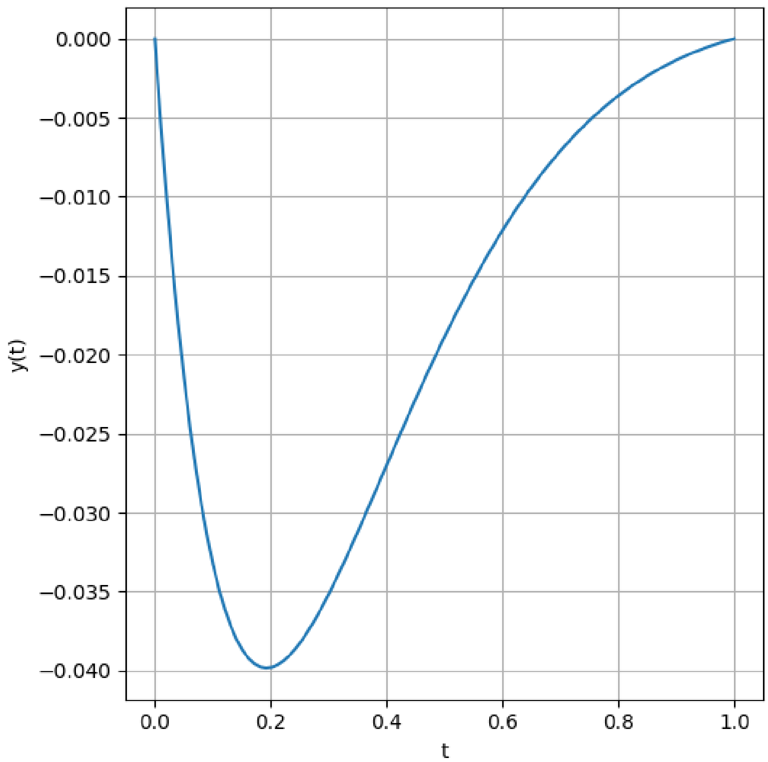

Figure 1: Deviation of a point $y(t)$ from the equilibrium position at time t