Relative Accelerometer Displacement During Sine Vibration

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In many aerospace applications, we need to monitor the relative displacement between two locations during sine vibrations to check clearance, estimate loads, etc. In NASTRAN FEA world, MPC (Multi-Point Constraint) equations or soft springs could be used to predict relative displacement between two nodes in a frequency and phase consistent manner. But how do we calculate the relative displacement between two accelerometers using test data?

Vibration test data acquisition systems provide FRFs (frequency response function) for acceleration and phase information at each accelerometer location as a function of frequency. At each frequency step, $\omega = 2\pi f$, acceleration responses in the time domain can be written as:

$$a_1(t) = A_1 \cos(\omega t - \varphi_1)$$

$$a_2(t) = A_2 \cos(\omega t - \varphi_2)$$

where A_1, φ_1, A_2, φ_2 are acceleration and phase amplitudes for locations 1 and 2 respectively. In harmonic vibrations, however, we know that displacements are also harmonic and have the following form:

$$d_1(t) = D_1 \cos(\omega t - \varphi_1)$$

$$d_2(t) = D_2 \cos(\omega t - \varphi_2)$$

where $D_1 = A_1/\omega^2$ and $D_2 = A_2/\omega^2$ are displacement amplitudes.

Relative displacement between the two locations will be:

$$\Delta(t) = d_2(t) - d_1(t)$$

since we are interested in the maximum relative displacement, we write:

$$\frac{d\Delta(t)}{dt} = 0$$

$$\rightarrow t^* = \frac{1}{\omega} \arctan\left(\frac{D_2 \sin(\varphi_2) - D_1 \sin(\varphi_1)}{D_2 \cos(\varphi_2) - D_1 \cos(\varphi_1)}\right) \tag{1}$$

substituting t^* back into $\Delta(t)$ equation we can numerically compute the max relative displacement. Alternatively, and using vectorial interpretation of the phase data, we can compute the relative displacement as:

$$\Delta^* = \sqrt{D_1^2 + D_2^2 - 2D_1 D_2 \cos(\varphi_2 - \varphi_1)} \quad (2)$$

Note that since relative displacement is also harmonic (the same period as displacements, accelerations, and the forcing function), maximum and minimum relative displacements are the same in absolute value.

Numerical Example

Consider accelerometers 1 & 2 measuring $A_1 = 10g$ and $A_2 = 8g$ respectively at 6Hz. Measured phases for these were $\varphi_1 = 30°$ and $\varphi_2 = 25°$ respectively. Figure 1 shows an overlay of these two harmonic responses in the time domain:

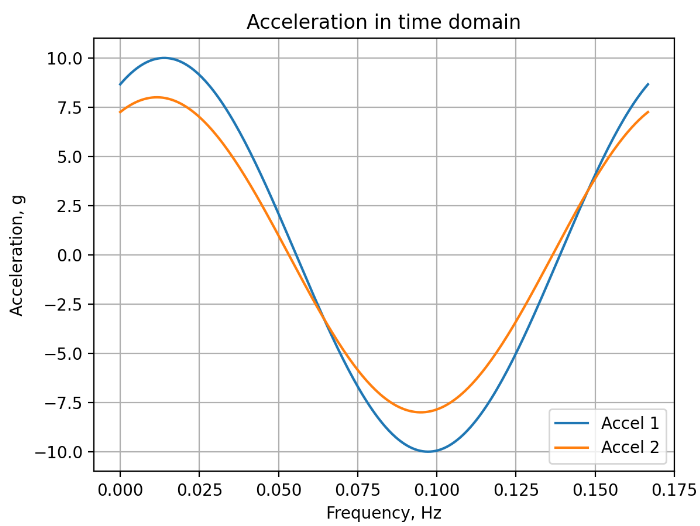

Figure 1: Acceleration in time domain

For these two accelerometers, we compute displacement amplitudes as:

$$D_1 = \frac{10 \cdot 386.1 in/s^2}{(2\pi \cdot 6/s)^2} = 2.717 in$$

$$D_2 = \frac{8 \cdot 386.1 in/s^2}{(2\pi \cdot 6/s)^2} = 2.173 in$$

Figure 2 shows displacements in the time domain:

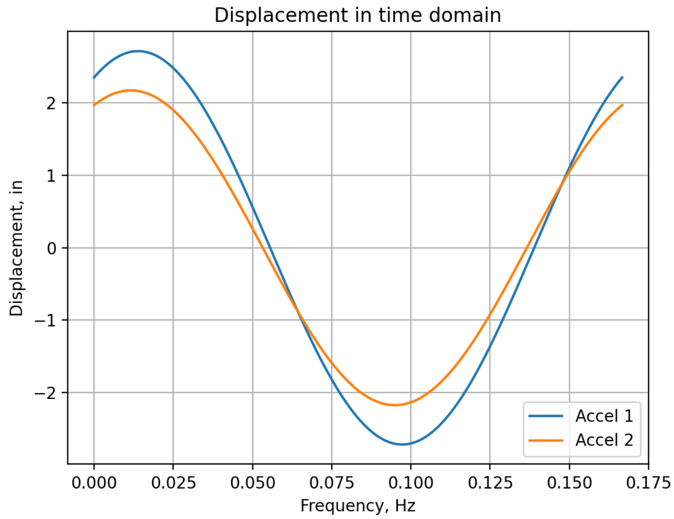

Figure 2: Displacement in time domain

Substituting numerical values in Equations 1 and 2 we get:

$$t^* = 0.022663s$$

and

$$\Delta = 0.583222in$$

If we were to calculate the relative displacement ignoring the phase difference, we would get $\Delta = D_1 - D_2 = 0.54334in$ which introduces 7% in error. Figure 3 compares results of the above formulas with numerical optimization using Python. In industry applications, we repeat the same process at each frequency step of the sine sweep. This provides valuable insights on structure behavior and dynamic loads.

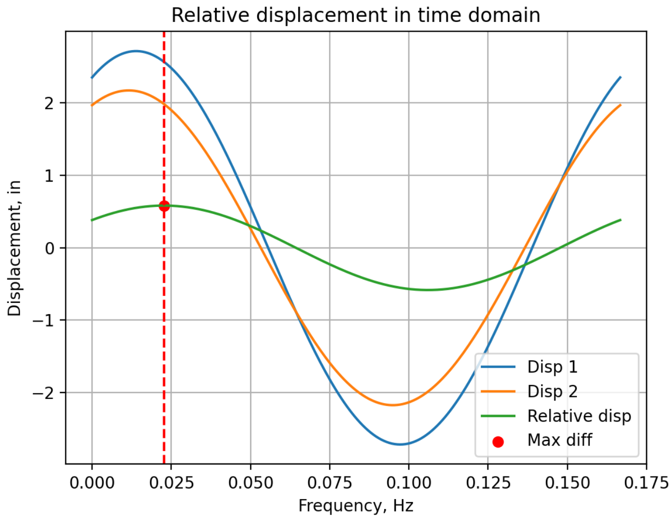

Figure 3: Relative displacement in time domain