Vibration Double Amplitude

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Vibration test shakers have a limitation on the displacement range they can base drive a test article. The full range of motion in the plus and minus directions is called double amplitude and typically varies from 0.5” to 2.0” depending on the test equipment/setup. As we know, harmonic vibration displacement is inversely proportional to excitation frequency squared. That is why we observe large displacement in the lower frequencies ($5Hz \approx 20Hz$) and displacement is almost unnoticeable to the naked eye in the higher frequencies ($> 1000Hz$).

Harmonic Vibrations

In harmonic vibration, shaker double amplitude displacement $D.A.$ at a given frequency $\omega = 2\pi f$ and acceleration is calculated as:

$$\ddot{x}\,(t) = A\cos\,(\omega t) \rightarrow \;x\,(t) = \frac{A}{\omega^2}\cos(\omega\;t)$$

$$D.A. = \frac{2A}{\omega^2} = \frac{2A}{4\pi^2 f^2}$$

In the Imperial Unit System, double amplitude becomes:

$$D.A. = \frac{19.56A}{f^2} \tag{1}$$

or alternatively:

$$A = 0.05112D.A.f^2 \tag{2}$$

where A is acceleration in g’s, $D.A.$ is in inches, and f is in Hz.

Example sine ramp-ups for different constant double amplitude limits are shown in Figure 1. This means that for a shaker setup with given D.A. limit, ramp-up slope cannot be above lines shown. Note that a constant $D.A$ ramp up has slope of $12.04dB/oct$.

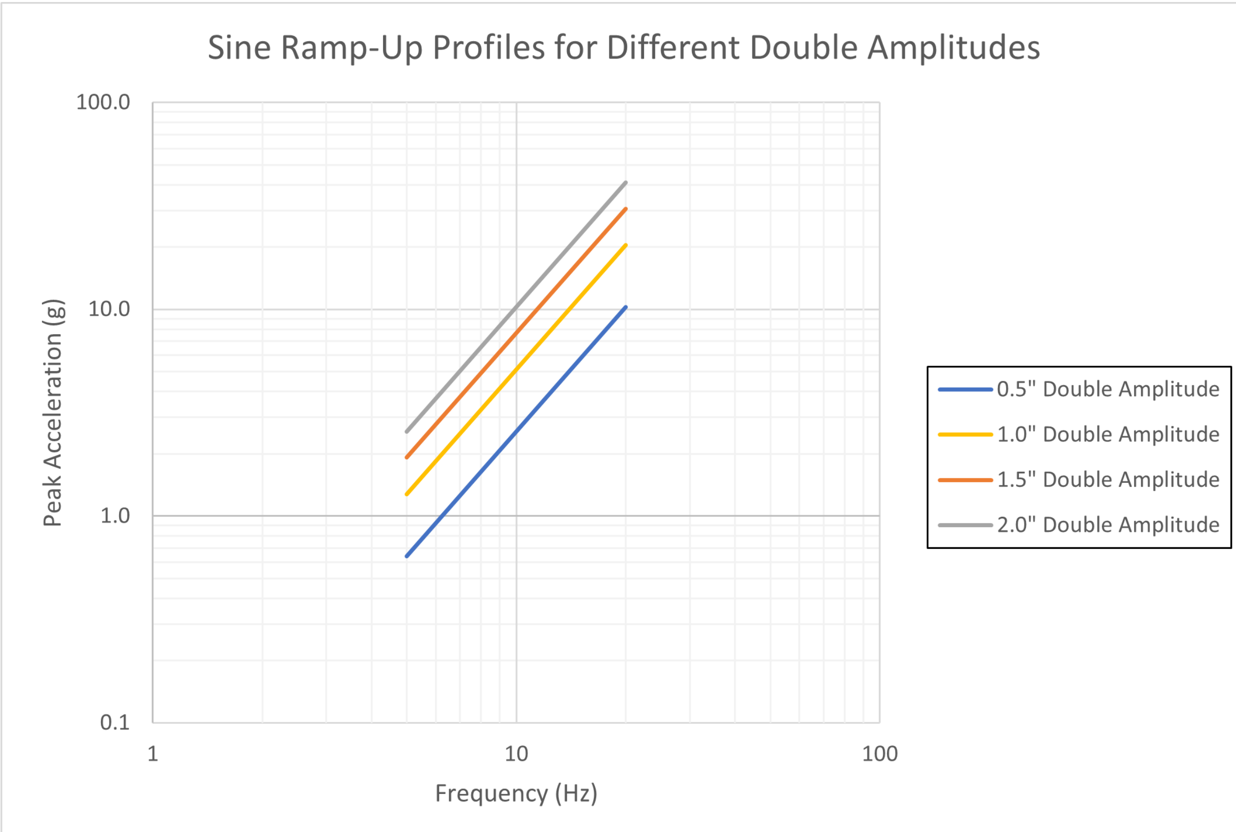

Figure 1: Sine Ramp Ups Corresponding to Different Constant Double Amplitude Limits

Another important note is that sine FRF plots should always be in a log-log system as that is how test controllers are setup to work. Plotting sine FRF data in a linear system could lead to incorrect conclusions on the sloped portions of the plot. We will discuss that in a separate article.

Random Vibrations

In most space applications, random vibration does not drive the shaker double amplitude requirement. However, we still need to know what is the maximum expected double amplitude displacement for a given random vibration profile.

Random vibration is specified in acceleration spectral density in g^2/Hz units vs frequency typically ranging from 20Hz to 2,000Hz. In ergodic random vibrations, maximum acceleration in the time domain is unknown but it is often assumed to be three times the rms (root mean square) acceleration. Since time average of a random acceleration is zero, the rms acceleration is the same as $1-\sigma$ standard deviation.

As we know, rms acceleration is calculated as the square root of the area under acceleration spectral density:

$$g_{rms} = \sqrt{\int_{f_1}^{f_2} W(f)\, df}$$

Slope of a log-log acceleration spectral density line from (f_1, W_1) to (f_1, W_2) in dB/oct (decibels per octaves) is defined as (note that $10\log(2) \approx 3.0$):

$$m = \frac{3.\log(W_2/W_1)}{\log(f_2/f_1)}$$

Given the slope m, equation of a spectral density line from (f_1, W_1) to (f, W) on a log-log scale can be written as:

$$W(f) = W_1.\left(\frac{f}{f_1}\right)^{\frac{m}{3}}$$

On the other hand, we know that displacement spectral density $\mathcal{D}$ in $\frac{in^2}{Hz}$ is written in terms of acceleration spectral density as follows ($g = 386.1 in^2/s$):

$$\mathcal{D}(f) = \frac{g^2 W(f)}{(2\pi f)^4}$$

This means that root mean displacement for one linear segment becomes:

$$d_{rms} = \sqrt{\int_{f_1}^{f_2} \mathcal{D} df} = \sqrt{\int_{f_1}^{f_2} \frac{g^2.W(f)}{(2.\pi.f)^4}}$$

As we previously discussed, maximum random vibration response is assumed to be the $3-\sigma$ value and since double amplitude accounts for the max total displacement in positive and negative directions, we will have:

$$D.A. = 3 \times 2 \times d_{rms}$$

Combining the above equations and after some algebraic simplifications we get the double amplitude formula for an N-segmented spectral density (f_1, W_1), (f_2, W_2), ..., (f_N, W_N) lines:

$$D.A. = 6.\sqrt{\sum_{i=1}^{i=N-1} \frac{1}{(2.\pi)^4}.\frac{3W_{i-1}}{(m_i-9).f_{i-1}^3}\left[\left(\frac{f_i}{f_{i-1}}\right)^{\frac{m_i}{3}-3} - 1\right]} \tag{3}$$

In the special case where for a line segment we have $m = 9$, we use L'Hôpital's rule to write:

$$\begin{aligned} D.A._i{}^2 &= \frac{1}{(2.\pi)^4}\frac{3W_{i-1}}{f_{i-1}^3}\lim_{m_i\to 9}\frac{\left[\left(\frac{f_i}{f_{i-1}}\right)^{\frac{m_i}{3}-3} - 1\right]}{(m_i-9)} \\ &= \frac{1}{(2.\pi)^4}\frac{3W_{i-1}}{f_{i-1}^3}\lim_{m_i\to 9}\left(\frac{f_i}{f_{i-1}}\right)^{\frac{m_i}{3}-3}\ln\left(\frac{f_i}{f_{i-1}}\right) \\ &= \frac{1}{(2.\pi)^4}\frac{3W_{i-1}}{f_{i-1}^3}\ln\left(\frac{f_i}{f_{i-1}}\right) \end{aligned}$$

Numerical Exmaple

Table 1 shows an example random vibration requirement for a spaceflight hardware along with its rms acceleration and displacements.

Table 1: Example Random Vibration Requirements.

$f(Hz)$	$W(g^2/Hz)$	$m(dB/oct)$	$\mathcal{D}(in^2/Hz)$
20	0.010	-	5.98e-6
50	0.100	7.54	1.53e-6
100	0.100	0.00	9.56e-8
150	0.200	5.13	3.78e-8
500	0.200	0.00	3.06e-10
2000	0.009	-6.71	5.38e-14
rms	$12.25g_{rms}$		$0.0108in_{rms}$

For this example, double amplitude becomes:

$$D.A. = 6 \times 0.0108in = 0.0646in$$

Figure 2 shows an overly of acceleration and displacement spectral densities. Note how quickly $\mathcal{D}$ diminishes in the higher frequencies (inversely proportional to f^4). This relationship explains why structures do not see high stresses in higher frequencies, e.g., $> 300Hz$. We will discuss this in more detail in another article.

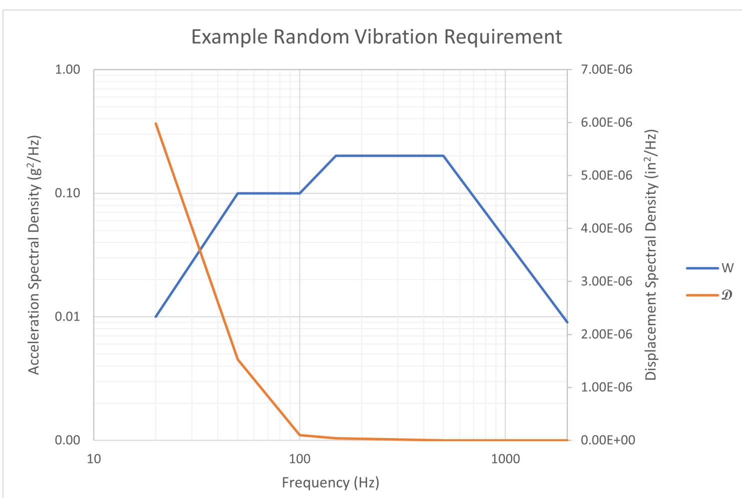

Figure 2: Example Random Vibration Requirement