

1 Approximated calculation using the definition of derivative

Problem 1: Calculate approximate value of

$$\sqrt[3]{30}. \quad (1)$$

Solution: For small Δx , it is true that $\Delta y \approx dy$, that is $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$, which follows from the definition of a derivative:

$$f'(x) = \frac{df}{dx} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (2)$$

Let's consider the function $f(x) = \sqrt[3]{x}$. Then we have

$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x \quad (3)$$

In our example, we have

$$\sqrt[3]{x + \Delta x} \approx \sqrt[3]{x} + \frac{d(\sqrt[3]{x})}{dx} \Delta x = \sqrt[3]{x} + \frac{1}{3\sqrt[3]{x^2}} \Delta x \quad (4)$$

In our case, $x + \Delta x = 30$. Assume that $x = 27$ and $\Delta x = 3$, then we have

$$\sqrt[3]{27 + 3} \approx \sqrt[3]{27} + \frac{1}{3\sqrt[3]{27^2}} 3 = 3 + \frac{3}{3 \cdot 9} = 3 + \frac{1}{9} = \frac{28}{9}. \quad (5)$$

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