1 Find an equation of the common chord of two intersected circles.

Problem 1: Without finding the intersection point of the circles, create an equation for their common chord.

$$\begin{aligned}(x-1)^2+(y+2)^2-18=0,\\ (x+3)^2+(y-1)^2-36=0.\end{aligned} \tag{1}$$

Solution: All quadrics (including the degenerate general chord whose equations need to be found) passing through the points P and Q belong to the bundle:

$$\lambda F+\mu G=0. \tag{2}$$

Let us take advantage of the fact that the general chord is a degenerate quadric, which means its invariants are equal to zero: $I_2=I_3=0$. Substituting Eqs. (1) into Eq. (2), we obtain

$$\lambda((x-1)^2+(y+2)^2-18)+\mu((x+3)^2+(y-1)^2-36)=0. \tag{3}$$

$$(\lambda+\mu)x^2+(\lambda+\mu)y^2+(-2\lambda+6\mu)x+(4\lambda-2\mu)y-13\lambda-26\mu=0. \tag{4}$$

$$I_2=\begin{vmatrix}\lambda+\mu & 0\\ 0 & \lambda+\mu\end{vmatrix}=0\leftrightarrow(\lambda+\mu)^2=0\leftrightarrow\lambda=-\mu \tag{5}$$

Let's choose $\lambda=1$, then we have $\mu=-1$. Substituting these values λ and μ into Eq. (4), we obtain the equation for common chord:

$$8x-6y-13=0. \tag{6}$$

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