Problem 1: Approximate the function $f(x) = 3^x$ on a segment $[-1, 1]$ with a polynomial function

$$Q(x) = a_0 + a_1 x + \ldots + a_m x^m = a_0 + a_1 x + a_2 x^2,$$

where a_i are constants.

Solution: To determine the coefficients, you need to draw up a system of 3 equations. This system can be obtained by equating the value of the function $f(x) = 3^x$ and $Q(x)$ at three points $x_0 = -1$, $x_1 = 0$, $x_2 = 1$:

$$\begin{aligned} &a_0 - a_1 + a_2 = 1/3, \\ &a_0 = 1, \\ &a_0 + a_1 + a_2 = 3. \end{aligned} \tag{1}$$

$$3^x \approx 1 + \frac{4}{3}x + \frac{2}{3}x^2. \tag{2}$$

When solving the problems of the theory of plates and shells by the variational methods of Ritz-Vlasov, Kontorovich, Bubnov-Galerkin, it becomes necessary to approximate the sought movement functions with polynomials. In this case, we select the system of functions to meet the boundary conditions. As approximating functions, most often various systems of functions given on a segment $[0, 1]$ are used, where $w(x)$ is positive continuous on a segment $[0, 1]$ function having a bounded and continuous derivative and satisfying the boundary conditions $w(0) = w(1) = 0$. In addition, the fundamental beam functions proposed by Vlasov are used as approximating functions.

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