Problem 1: Determine the stress of the steel and copper parts of the stepped rod when the temperature rises by $\Delta T = 30°\text{C}$. Dimensions are $l_{st} = 1$ m, $l_{cu} = 1.4$ m, gap $\delta = 0.0003$ m, $A_{st} = 0.001$ m^2, $A_{cu} = 0.002$ m^2; moduli of elasticity are $E_{st} = 2 \cdot 10^6$ kgf/cm^2 $= 2 \cdot 10^{10}$ kgf/m^2 $= 196.133$ GPa, and $E_{cu} = 1 \cdot 10^6$ kgf/cm^2 $= 1 \cdot 10^{10}$ kgf/m^2 $= 98.0665$ GPa. Thermal expansion coefficients are $\alpha_{st} = 12.5 \cdot 10^{-6}$ 1/C° and $\alpha_{cu} = 16.5 \cdot 10^{-6}$ 1/C°.

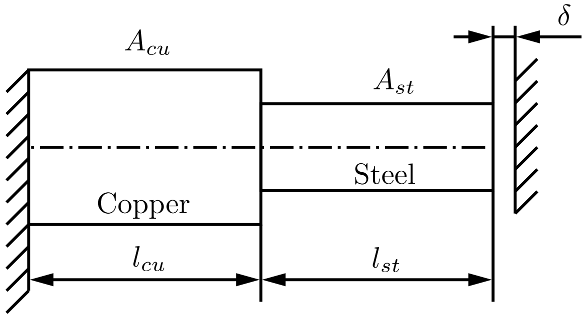

Figure 1: Stepped rod.

Solution: If structural elements are deprived of free deformation, then when the temperature changes, the so-called thermal forces, and the corresponding thermal stresses arise. Such a phenomenon, for example, is observed during the heating or cooling of statically indeterminate structures. The simplest example of such a design is a rod fixed on rigid, unyielding supports. The stresses arising in the cross sections of a homogeneous rod fixed at the ends when the temperature changes are determined by the formula

$$\sigma_t = \pm E\alpha\Delta T, \tag{1}$$

where α is the thermal expansion coefficients, ΔT is the change in temperature against the initial value, E is the modulus of elasticity. Stresses are positive (tensile) if the temperature drops and negative (compressive) if the temperatures rise. If there is a gap between one of the ends of the rod and a rigid support, the thermal stress is defined by

$$\sigma_t = -E\left(\alpha\Delta T - \frac{\delta}{l}\right), \tag{2}$$

where l the rod length.

In the case of a stepped rod with pinched ends, the force arising from a temperature change is determined by

$$N_t = \pm\frac{\Delta T\sum_{i=1}^{n}\alpha_i l_i}{\sum_1^n \frac{l_i}{E_i A_i}}, \tag{3}$$

and with the presence of a gap the force is given by

$$N_t = -\frac{\Delta T\sum_{i=1}^{n}\alpha_i l_i - \delta}{\sum_1^n \frac{l_i}{E_i A_i}}, \tag{4}$$

where α_i is the thermal expansion coefficient of the material of step i; l_i is the length of step i; A_i is the cross-sectional area of step i; E_i is the modulus of elasticity of step i. The temperature stress in the cross section of any step is equal to

$$\sigma_i = \frac{N_i}{A_i}. \tag{5}$$

For the rod with two steps we have

$$N_t = -\frac{\Delta T(\alpha_{st} \cdot l_{st} + \alpha_{cu} \cdot l_{cu}) - \delta}{\frac{l_{st}}{E_{st}A_{st}} + \frac{l_{cu}}{E_{cu}A_{cu}}} = \\ = -\frac{30(12.5 \cdot 10^{-6} \cdot 1.0 + 16.5 \cdot 10^{-6} \cdot 1.40) - 0.0003}{\frac{1.0}{2 \cdot 10^{10} \cdot 0.001} + \frac{1.4}{1 \cdot 10^{10} \cdot 0.002}} = -6400 \text{ kgf}, \tag{6}$$

The stresses in the cross sections are equal to

1. in steel part:
$$\sigma_{st} = \frac{N_t}{A_{st}} = -\frac{6400}{0.001} = -6400000 \text{ kgf/m}^2 = -62.76256 \text{ MPa}, \tag{7}$$

2. in copper part:
$$\sigma_{cu} = \frac{N_t}{A_{cu}} = -\frac{6400}{0.002} = -3200000 \text{ kgf/m}^2 = -31.38128 \text{ MPa}. \tag{8}$$

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