Problem 1: To increase the axial moment of inertia (flexural rigidity) of a beam of square cross section ($a \times a$), plates of rectangular cross-section ($b \times h$) are welded on top and bottom of the beam (Figure 1). Determine how many times the weight of the beam will increase if the axial moment of inertia doubles ($I_x^h = 2I_x^c$) and the elastic section modulus remains unchanged ($S_h = S_c$).

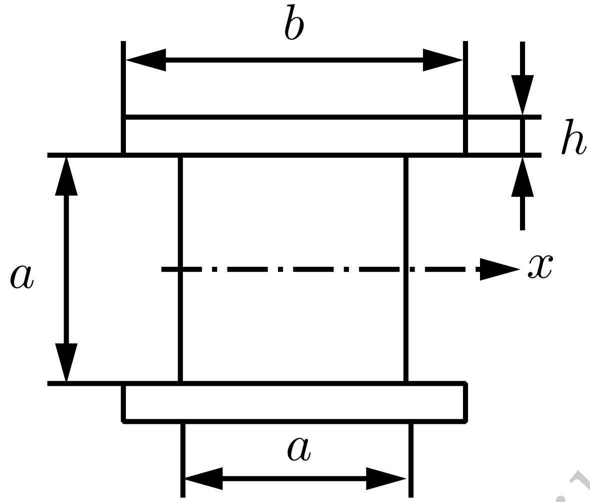

Figure 1: Beam cross-section.

Solution: The increase in the weight of the beam is determined by the ratio of two areas $A_h = A_c$, where $A_c = a^2$ and $A_h = a^2 + 2bh$. To define a parameter, we use a condition $S_h = S_c$, i.e.

$$\begin{aligned} S_c &= \frac{I_x^c}{y_{max}^c}, \\ S_h &= \frac{I_x^h}{y_{max}^h}. \end{aligned} \tag{1}$$

Therefore, we have

$$\frac{I_x^c}{y_{max}^c} = \frac{I_x^h}{y_{max}^h}, \tag{2}$$

or

$$\frac{I_x^c}{y_{max}^c} = \frac{2I_x^c}{y_{max}^h}. \tag{3}$$

We obtain

$$y_{max}^h = 2y_{max}^c = 2\left(\frac{a}{2}\right), \tag{4}$$

from where we get the following

$$h = \frac{a}{2}. \tag{5}$$

From the condition $I_x^h = 2I_x^c$, we obtain

$$\frac{a^4}{12} + 2(bh)\left(\frac{3}{4}a\right)^2 = 2\frac{a^4}{12}. \tag{6}$$

From this equation we can obtain

$$b = \frac{4}{27}a, \tag{7}$$

and

$$A_h = a^2 + 2bh = a^2 + 2\left(\frac{4}{27}a\right)\left(\frac{a}{2}\right) = \frac{31}{27}a^2. \tag{8}$$

Therefore, the sought ratio of areas is

$$\frac{A_h}{A_c} = \frac{31}{27}. \tag{9}$$

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